k-Limited Erasing Performed by Regular-Controlled Context-Free Grammars

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Outline

1. Light introduction to regulated rewriting

2. Motivation and problem description

3. Results

From Context-Free Grammars. . .

Consider the following context-free grammar:

$$S \rightarrow ABC \qquad\qquad A \rightarrow \varepsilon$$
$$A \rightarrow aA \qquad\qquad B \rightarrow \varepsilon$$
$$B \rightarrow bB \qquad\qquad C \rightarrow \varepsilon$$
$$C \rightarrow cC$$

From Context-Free Grammars. . .

Consider the following context-free grammar:

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$R = \{1\}\{234\}^*\{567\}$

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$L = \{a^n b^n c^n \mid n \geq 0\}$

Open Problem

Can any
regular-controlled context-free grammar with erasing rules
be transformed into an equivalent
regular-controlled context-free grammar without erasing rules?

Main Result

Condition Erasure of nonterminals in a *k*-limited way, $k \geq 0$.

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Partial solution of the problem.

Thank you for your attention!