On Nondeterminism in Programmed Grammars

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Example

$$
\begin{aligned}
&(1\colon S \to ABC, \{2,5\})\\
&(2\colon A \to aA, \{3\})\\
&(3\colon B \to bB, \{4\})\\
&(4\colon C \to cC, \{2,5\})\\
&(5\colon A \to a, \{6\})\\
&(6\colon B \to b, \{7\})\\
&(7\colon C \to c, \emptyset)
\end{aligned}
$$

$$
\begin{aligned}
(S, 1) &\Rightarrow (ABC, 2)\\
&\Rightarrow (aABC, 3)\\
&\Rightarrow (aAbBC, 4)\\
&\Rightarrow (aAbBcC, 5)\\
&\Rightarrow (aabBcC, 6)\\
&\Rightarrow (aabbcC, 7)\\
&\Rightarrow (aabbcc, \bot)
\end{aligned}
$$

$$
\begin{tikzcd}
 & 2 \arrow[r] & 3 \arrow[r] & 4 \arrow[ll, bend right] \arrow[ddll] \\
1 \arrow[ur] \arrow[dr] & & & \\
 & 5 \arrow[r] & 6 \arrow[r] & 7
\end{tikzcd}
$$

$$L(G) = \{a^n b^n c^n \mid n \geq 1\}$$

Three types of nondeterminism:

1. Which rule should be chosen as the first one?
2. Which occurrence of a nonterminal should be rewritten?
3. Which successor should be chosen?

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What has been studied:

- at most one successor [Bordihn, Holzer 2006]
- at most two successors [Bordihn, Holzer 2006]
- graphs from various classes [BBDDFILLN, 2006]

Central topic of our paper:

- continue the study of the role of nondeterminism in programmed grammars

Motivation:

- theoretical: normal forms
- practical: parsing

Results: Number of Nondeterministic Rules

Definition

Nondeterministic rule: a rule with more than one successor.

Example

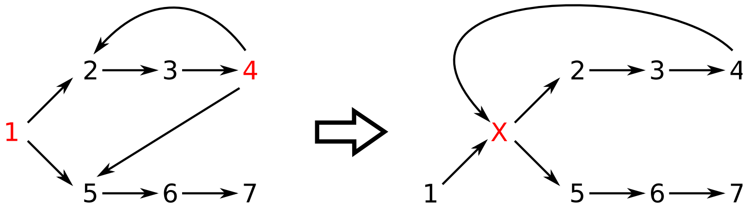
Nondeterministic rule: $(1\colon S \rightarrow ABC, \{2, 5\})$
Deterministic rule: $(2\colon A \rightarrow aA, \{3\})$

Definition

Nondeterministic rule: a rule with more than one successor.

Result

Every programmed grammar can be transformed into an equivalent programmed grammar with only a single nondeterministic rule.



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For the original rule $(1: S \rightarrow ABC, \{2, 5\})$, we introduce the following rules:

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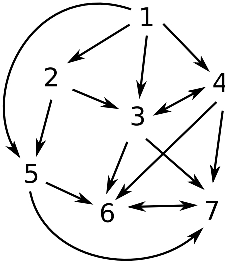
- $(1\colon S \to \langle 1 \rangle \$, \{X\})$
- $(X\colon \$ \to \varepsilon, \{\dots, r, s, \dots\})$
- $(r\colon \langle 1 \rangle \to ABC, \{2\})$
- $(s\colon \langle 1 \rangle \to ABC, \{5\})$

Results: Overall Nondeterminism



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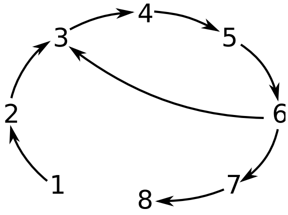

- The first grammar is harder to analyze/parse.
- Nondeterministic rules increase complexity.
- No problem with deterministic rules.

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Formalization

Overall nondeterminism: the sum of the number of successors of each nondeterministic rule.

Example

Consider $\{a^n b^n c^n \mid n \geq 1\}$.

$(1 : S \to ABC, \{2, 5\})$
$(2 : A \to aA, \{3\})$
$(3 : B \to bB, \{4\})$
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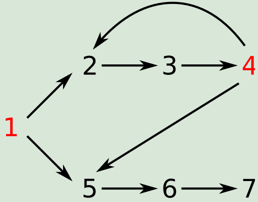

Overall nondeterminism: 4

$(1 : S \to ABC, \{2\})$
$(2 : A \to aA, \{3\})$
$(3 : B \to bB, \{4\})$
$(4 : C \to cC, \{2, 5\})$
$(5 : A \to \varepsilon, \{6\})$
$(6 : B \to \varepsilon, \{7\})$
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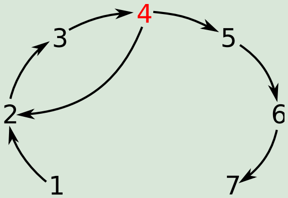

Overall nondeterminism: 2

Results: Overall Nondeterminism

Result

We cannot limit overall nondeterminism without losing generality.

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Consider the alphabet $\Sigma = \{a_1, a_2, \ldots, a_n\}$. Define

$$L_n = \bigcup_{i=1}^{n} \{a_i\}^+$$

L_n has overall nondeterminism $n + 1$.

Open Problems

- Proof of the second result without a growing alphabet?
- Programmed grammars with appearance checking?
- Programmed grammars without ε-rules?
- Programmed grammars with leftmost derivations?

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Discussion