

An Infinite Hierarchy of Language Families Resulting from n -limited Programmed Grammars

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Programmed Grammar

A *programmed grammar* is a quadruple

$$G = (N, T, S, P),$$

where

- N is an alphabet of *nonterminals*;
- T is an alphabet of *terminals*;
- S is the starting nonterminal;
- P is a finite set of productions of the form $(r : A \rightarrow v, \sigma(r))$.



Example

(1 : $S \rightarrow ABC, \{2, 5\}$)

(2 : $A \rightarrow aA, \{3\}$)

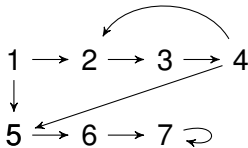
(3 : $B \rightarrow bB, \{4\}$)

(4 : $C \rightarrow cC, \{2, 5\}$)

(5 : $A \rightarrow a, \{6\}$)

(6 : $B \rightarrow b, \{7\}$)

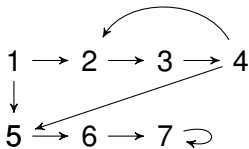
(7 : $C \rightarrow c, \{7\}$)



Example

- (1 : $S \rightarrow ABC, \{2, 5\}$)
- (2 : $A \rightarrow aA, \{3\}$)
- (3 : $B \rightarrow bB, \{4\}$)
- (4 : $C \rightarrow cC, \{2, 5\}$)
- (5 : $A \rightarrow a, \{6\}$)
- (6 : $B \rightarrow b, \{7\}$)
- (7 : $C \rightarrow c, \{7\}$)

$S \Rightarrow ABC$ [1]
 $\Rightarrow aABC$ [2]
 $\Rightarrow aAbBC$ [3]
 $\Rightarrow aAbBcC$ [4]
 $\Rightarrow aabBcC$ [5]
 $\Rightarrow aabbcc$ [6]
 $\Rightarrow aabbcc$ [7]



$$L(G) = \{a^n b^n c^n : n \geq 1\}$$



Generative Power



Leftmost Derivations

Idea: At each step of a derivation the leftmost occurrence of a nonterminal has to be rewritten.

Example: $(r : A \rightarrow v, \sigma(r))$
 $wA_1xA_2yA_3z \Rightarrow wvxA_2yA_3z [r]$ (only!)

Problem: We decrease the generative power of programmed grammars to **CF**.



n -limited Derivations

Idea: At each step of a derivation **at most the n th** occurrence (from the left) of a nonterminal has to be rewritten, where $n \geq 1$.

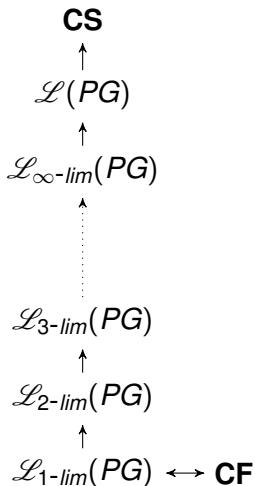
Example: $(r : A \rightarrow v, \sigma(r))$

$$\begin{aligned} x_0 A_1 x_1 A_2 x_2 \dots A_n x_n \dots A_{n+1} x_{n+1} \dots A_h x_h &\Rightarrow \\ x_0 v x_1 A_2 x_2 \dots A_n x_n \dots A_{n+1} x_{n+1} \dots A_h x_h [r] &\text{ or} \\ x_0 A_1 x_1 v x_2 \dots A_n x_n \dots A_{n+1} x_{n+1} \dots A_h x_h [r] &\text{ or} \\ &\vdots \\ x_0 A_1 x_1 A_2 x_2 \dots v x_n \dots A_{n+1} x_{n+1} \dots A_h x_h [r]. \end{aligned}$$

Question: How does this affect the generative power?



Infinite Hierarchy of Language Families



End of presentation, thank you for your attention!

Any questions?